Different useful aspects of hierarchical quantum communication

Chitra Shukla[1] * Anirban Pathak[1] †

[1] *Jaypee Institute of Information Technology, A-10, Sector-62, Noida, UP-201307, India.*

Abstract. A few schemes of hierarchical quantum information splitting (HQIS) is proposed in recent past. Some generalizations of the existing schemes of HQIS are proposed and explicit examples of HQIS using 8-qubit cluster state and a few 4-qubit states are provided. It is shown that the states employed for HQIS can also be used for probabilistic hierarchical quantum information splitting and hierarchical quantum secret sharing (HQSS). Further, some examples of practical situations where hierarchical quantum communication would be of use, are reported.

Keywords: Hierarchical quantum communication, QIS, QSS.

1 Introduction

Many schemes of multi-party quantum teleportation (symmetric quantum information splitting) have been proposed in last two decades and that led to a set of interesting applications like the controlled teleportation and the quantum information splitting (QIS) schemes. Other aspects of quantum communication like probabilistic QIS, quantum secret sharing (QSS), etc. can be viewed as applications of QIS. Recently, Wang *et al.* [1]-[3] introduced the concept of asymmetric quantum information splitting in which a boss (Alice) distributes a quantum state among several agents who are spatially separated. The agents are graded according to their power to recover the quantum state sent by Alice. A high power agent does not require the help of all other agents to reconstruct the quantum state, whereas a low power agent can reconstruct it iff all other agents cooperate with him. Thus, there is a hierarchy among the agents, which is why the scheme is referred to as a hierarchical QIS (HQIS) scheme. We have generalized the idea of Wang *et al.* and have investigated the possibility of HQIS using an arbitrary $(n+1)$-qubit entangled state [4]. We have also shown that HQIS is possible for different classes of 4-qubit entangled quantum states and 8-qubit cluster state. We have further generalized our scheme to introduce schemes for probabilistic HQIS, and hierarchical quantum secret sharing (HQSS). Due to limitation of space only results related to HQIS using 8-qubit cluster state is shown below.

2 A generalized approach to perfect HQIS

Let us start with an entangled $(n+1)$-qubit state of the form

$$|\psi_c\rangle = \frac{1}{\sqrt{2}}[|0\rangle|\psi_0\rangle + |1\rangle|\psi_1\rangle], \quad (1)$$

where $|\psi_0\rangle$ and $|\psi_1\rangle$ are arbitrary n qubit states that are orthogonal to each other. The subscript c stands for channel. The first qubit of $|\psi_c\rangle$ is with Alice and the rest are with n agents, say $\mathrm{Bob}_{1\cdots n}$. Alice wishes to teleport (share) among her agents a general one qubit state

$$|\psi_s\rangle = \frac{1}{\sqrt{1+|\lambda|^2}}(|0\rangle + \lambda|1\rangle). \quad (2)$$

So the combined state of Alice and her agents is

$$\begin{array}{l} |\psi_s\rangle \otimes |\psi_c\rangle = \\ \frac{1}{\sqrt{1+|\lambda|^2}}\left(|0\rangle + \lambda|1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left[|0\rangle|\psi_0\rangle + |1\rangle|\psi_1\rangle\right] \\ = \frac{1}{\sqrt{2(1+|\lambda|^2)}}[(|00\rangle|\psi_0\rangle + |01\rangle|\psi_1\rangle) + \lambda(|10\rangle|\psi_0\rangle + |11\rangle|\psi_1\rangle)] \\ = \frac{1}{2\sqrt{(1+|\lambda|^2)}}\left[|\psi^+\rangle\left(|\psi_0\rangle + \lambda|\psi_1\rangle\right) + |\psi^-\rangle\left(|\psi_0\rangle - \lambda|\psi_1\rangle\right)\right. \\ \left. + |\phi^+\rangle\left(|\psi_1\rangle + \lambda|\psi_0\rangle\right) + |\phi^-\rangle\left(|\psi_1\rangle - \lambda|\psi_0\rangle\right)\right], \end{array} \quad (3)$$

where $|\psi^\pm\rangle$ and $|\phi^\pm\rangle$ are Bell states given by $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$, $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$.

Now Alice performs a Bell measurement on the first 2 qubits. From (3) we can see that after the Bell measurement of Alice the combined states of all the n agents reduces to $|\Psi^\pm\rangle = \frac{|\psi_0\rangle \pm \lambda|\psi_1\rangle}{\sqrt{1+|\lambda|^2}}$ or $|\Phi^\pm\rangle = \frac{|\psi_1\rangle \pm \lambda|\psi_0\rangle}{\sqrt{1+|\lambda|^2}}$. The complete relation between the measurement outcome of Alice and the combined state of the agents is given in Table 1 of [4]. This provides a basic structure to study possibilities of HQIS using different quantum states. A specific example is provided below using a 8-qubit cluster state.

2.1 A special case: $|\psi_c\rangle$ is a 8-qubit cluster state ($|C_8\rangle$)

Let us choose a 8-qubit cluster state as channel and the first photon is with Alice and the rest are with Bob_1 to Bob_7. Thus

$$\begin{array}{l} |\psi_c\rangle = \\ \frac{1}{2}[|00000000\rangle + |00001111\rangle + |11110000\rangle - |11111111\rangle_{1\cdots 8}] \\ = \frac{1}{\sqrt{2}}[|0\rangle_1|\psi_0\rangle_{2\cdots 8} + |1\rangle_1|\psi_1\rangle_{2\cdots 8}], \end{array} \quad (4)$$

where $|\psi_0\rangle = \frac{1}{\sqrt{2}}[|0000000\rangle + |0001111\rangle]_{2\cdots 8}$ and $|\psi_1\rangle = \frac{1}{\sqrt{2}}[|1110000\rangle - |1111111\rangle]_{2\cdots 8}$.

Now after Alice's Bell measurement on the first two qubits if Alice's measurement outcome is $|\psi^\pm\rangle$ then the combined state of $\mathrm{Bob}_1 - \mathrm{Bob}_7$ collapses to $|\Psi^\pm\rangle_{2\cdots 8}$ which is written as (5) and can be rearranged as (6)

*chitrashukla07@gmail.com

†anirban.pathak@jiit.ac.in

Table 1: Relation among the Bell measurement outcomes of Alice and $\mathrm{Bob}_{2\cdots7}$ and the unitary operations to be applied by Bob_1.

Alice outcome	$\mathrm{Bob}_{2,3}$ outcome	$\mathrm{Bob}_{4,5,6,7}$ outcome	Bob_1 operation
$\|\psi^+\rangle$, $\|\psi^-\rangle$	$\|\psi^+\rangle$	$\|0000\rangle$	I, Z
$\|\psi^+\rangle$, $\|\psi^-\rangle$	$\|\psi^-\rangle$	$\|0000\rangle$	Z, I
$\|\psi^+\rangle$, $\|\psi^-\rangle$	$\|\psi^+\rangle$	$\|1111\rangle$	Z, I
$\|\psi^+\rangle$, $\|\psi^-\rangle$	$\|\psi^-\rangle$	$\|1111\rangle$	I, Z
$\|\phi^+\rangle$, $\|\phi^-\rangle$	$\|\psi^+\rangle$	$\|0000\rangle$	X, iY
$\|\phi^+\rangle$, $\|\phi^-\rangle$	$\|\psi^-\rangle$	$\|0000\rangle$	iY, X
$\|\phi^+\rangle$, $\|\phi^-\rangle$	$\|\psi^+\rangle$	$\|1111\rangle$	iY, X
$\|\phi^+\rangle$, $\|\phi^-\rangle$	$\|\psi^-\rangle$	$\|1111\rangle$	X, iY

$$|\Psi^{\pm}\rangle_{2\cdots8} = \frac{1}{\sqrt{2(1+|\lambda|^2)}}[|0000000\rangle + |0001111\rangle \pm\lambda(|1110000\rangle - |1111111\rangle)]_{2\cdots8}. \quad (5)$$

$$|\Psi^{\pm}\rangle_{2\cdots8} = \frac{1}{2\sqrt{(1+|\lambda|^2)}}[|0\psi^+0000\rangle + |0\psi^-0000\rangle +|0\psi^+1111\rangle + |0\psi^-1111\rangle \pm \lambda(|1\psi^+0000\rangle -|1\psi^-0000\rangle - |1\psi^+1111\rangle + |1\psi^-1111\rangle)]_{2\cdots8}. \quad (6)$$

If the agents decide that Bob_1 will recover the quantum state sent by Alice, then we can decompose (6) as

$$\begin{aligned}|\Psi^{\pm}\rangle_{2\cdots8} = & \frac{1}{2\sqrt{(1+|\lambda|^2)}}[(|0\rangle \pm \lambda|1\rangle)_2|\psi^+0000\rangle_{3\cdots8} \\ & +(|0\rangle \mp \lambda|1\rangle)_2|\psi^-0000\rangle_{3\cdots8} \\ & +(|0\rangle \mp \lambda|1\rangle)_2|\psi^+1111\rangle_{3\cdots8} \\ & +(|0\rangle \pm \lambda|1\rangle)_2|\psi^-1111\rangle_{3\cdots8}]\end{aligned} \quad (7)$$

From (7) if Alice's Bell measurement outcome is $|\psi^+\rangle(|\psi^-\rangle)$ then Bob_1 can reconstruct the unknown state by applying an unitary operator $I(Z)$ with the collaboration of Bob_{23} (with Bell measurement outcome $|\psi^+\rangle$), and one of Bob_{4567} (with computational measurement outcome $|0\rangle$). As the measurement outcomes of Bob_{4567} are the same, so the communications from one of them, Bob_{23} and Alice would be sufficient for Bob_1 to reconstruct the unknown state. Similarly, Bob_1 can recover the secret state for all other cases of (7) and also if Alice's measurement outcome is $|\phi^{\pm}\rangle$, for which operations applied by Bob_1 are shown in Table 1.

Now if agents allows Bob_7 to recover the unknown state then (5) can be rearranged as (8) and decomposed as (9)

$$\begin{aligned}&|\Psi^{\pm}\rangle_{2\cdots8} = \\ &\frac{1}{4\sqrt{(1+|\lambda|^2)}}[(|\psi^+\rangle + |\psi^-\rangle)(|\psi^+\rangle + |\psi^-\rangle)(|\psi^+\rangle + |\psi^-\rangle)|0\rangle \\ &+(|\psi^+\rangle + |\psi^-\rangle)(|\phi^+\rangle + |\phi^-\rangle)(|\psi^+\rangle - |\psi^-\rangle)|1\rangle \\ &\pm\lambda\{(|\psi^+\rangle - |\psi^-\rangle)(|\phi^+\rangle - |\phi^-\rangle)(|\psi^+\rangle + |\psi^-\rangle)|0\rangle \\ &-(|\psi^+\rangle - |\psi^-\rangle)(|\psi^+\rangle - |\psi^-\rangle)(|\psi^+\rangle - |\psi^-\rangle)|1\rangle\}]_{2\cdots8}.\end{aligned} \quad (8)$$

$$\begin{aligned}|\Psi^{\pm}\rangle_{2\cdots8} = & \frac{1}{4\sqrt{(1+|\lambda|^2)}}[|\psi^+\psi^+\psi^+\rangle_{2\cdots7}(|0\rangle \mp \lambda|1\rangle)_8 \\ &+|\psi^+\psi^-\psi^+\rangle_{2\cdots7}(|0\rangle \pm \lambda|1\rangle)_8 + |\psi^-\psi^+\psi^+\rangle_{2\cdots7}(|0\rangle \pm \lambda|1\rangle)_8 \\ &+|\psi^-\psi^-\psi^+\rangle_{2\cdots7}(|0\rangle \mp \lambda|1\rangle)_8 + |\psi^+\psi^+\psi^-\rangle_{2\cdots7}(|0\rangle \pm \lambda|1\rangle)_8 \\ &+|\psi^+\psi^-\psi^-\rangle_{2\cdots7}(|0\rangle \mp \lambda|1\rangle)_8 + |\psi^-\psi^+\psi^-\rangle_{2\cdots7}(|0\rangle \mp \lambda|1\rangle)_8 \\ &+|\psi^-\psi^-\psi^-\rangle_{2\cdots7}(|0\rangle \pm \lambda|1\rangle)_8 + |\psi^+\phi^+\psi^+\rangle_{2\cdots7}(|1\rangle \pm \lambda|0\rangle)_8 \\ &+|\psi^+\phi^-\psi^+\rangle_{2\cdots7}(|1\rangle \mp \lambda|0\rangle)_8 + |\psi^-\phi^+\psi^+\rangle_{2\cdots7}(|1\rangle \mp \lambda|0\rangle)_8 \\ &+|\psi^-\phi^-\psi^+\rangle_{2\cdots7}(|1\rangle \pm \lambda|0\rangle)_8 - |\psi^+\phi^+\psi^-\rangle_{2\cdots7}(|1\rangle \mp \lambda|0\rangle)_8 \\ &-|\psi^+\phi^-\psi^-\rangle_{2\cdots7}(|1\rangle \pm \lambda|0\rangle)_8 - |\psi^-\phi^+\psi^-\rangle_{2\cdots7}(|1\rangle \pm \lambda|0\rangle)_8 \\ &-|\psi^-\phi^-\psi^-\rangle_{2\cdots7}(|1\rangle \mp \lambda|0\rangle)_8].\end{aligned} \quad (9)$$

According to (9) if Alice's Bell measurement outcome and outcomes of joint (Bell) measurements of Bob_{12}, Bob_{34} and Bob_{56} are communicated to Bob_7 then he can reconstruct the unknown state by applying appropriate unitary operators. As communications from all Bobs and Alice are required by Bob_7, but the same was not required by Bob_1 so there exists a hierarchy. Similar conclusions can be obtained when outcome of Bell measurement of Alice is $|\phi^{\pm}\rangle$.

3 Conclusions

Wang *et al.*'s idea of asymmetric quantum information splitting is generalized to provide a general framework to study HQIS. Proposed scheme is modified to yield protocols of HQSS and probabilistic HQIS which are not elaborated here due to space limitations. Detail of these protocols can be found in [4]. The generalizations reported here are expected to find applications in many reallife scenarios. Furthermore, the approach adopted here can be easily used to study the possibilities of observing HQIS, probabilistic HQIS and HQSS in other quantum states.

Acknowledgment: A. P. thanks DST, India for support provided through project No. SR/S2/LOP-0012/2010.

References

[1] X.-W. Wang *et al.* Hierarchical quantum-information splitting. Opt. Comm. 283: 1196-1199, 2010.

[2] X.-W. Wang *et al.* Multiparty hierarchical quantum-information splitting. J. Phys. B 44: 035505, 2011.

[3] X.-W. Wang *et al.* Hierarchical Quantum Information Splitting with Six-Photon Cluster States. Int. J. Theor. Phys. 49: 2691-2697, 2010.

[4] C. Shukla and A. Pathak. Hierarchical quantum communication. Phys. Lett. A 377: 1337-1344, 2013.